OF THE GEOMETRICAL SPIRIT

Blaise Pascal

Of the Geometrical Spirit

Blaise Pascal

Translated by O.W. Wright

Rows Collection

Text is from *The Harvard Classics. 1909–14.*

Rendered into HTML by Steve Thomas .

Last updated Wednesday, December 17, 2014 at 14:20.

Of the Geometrical Spirit

We may have three principal objects in the study of truth: one to discover it when it is sought; another to demonstrate it when it is possessed; and a third, to discriminate it from the false when it is examined.

I do not speak of the first; I treat particularly of the second, and it includes the third. For if we know the method of proving the truth, we shall have, at the same time, that of discriminating it, since, in examining whether the proof that is given of it is in conformity with the rules that are understood, we shall know whether it is exactly demonstrated.

Geometry, which excels in these three methods, has explained the art of discovering unknown truths; this it is which is called *analysis,* and of which it would be useless to discourse after the many excellent works that have been written on it.

That of demonstrating truths already found, and of elucidating them in such a manner that the proof of them shall be irresistible, is the only one that I wish to give; and for this I have only to explain the method which geometry observes in it; for she teaches it perfectly by her examples, although she may produce no discourse on it. And since this art consists in two principal things, the one in proving each proposition by itself, the other in disposing all the propositions in the best order, I shall make of it two sections, of which the one will contain the rules for the conduct of geometrical, that is, methodical and perfect demonstrations; and the second will comprehend that of geometrical, that is, methodical and complete order: so that the two together will include all that will be necessary to direct reasoning, in proving and discriminating truths, which I design to give entire.

Section first — Of the method of geometrical, that is, of methodical and perfect demonstrations.

I cannot better explain the method that should be preserved to render demonstrations convincing, than by explaining that which is observed by geometry.

But it is first necessary that I should give the idea of a method still more eminent and more complete, but which mankind could never attain; for what exceeds geometry sur-passes us; and, nevertheless, something must be said of it, although it is impossible to practise it.[1]

This true method, which would form demonstrations in the highest excellence, if it were possible to arrive at it, would consist in two principal things: the one, in employing no term the meaning of which had not first been clearly explained; the other, in never advancing any proposition which could not be demonstrated by truths already known; that is, in a word, in defining every term, and in proving every proposition. But to follow the same order that I am explaining, it is necessary that I should state what I mean by *definition.*

The only definitions recognized in geometry are what the logicians call *definitions of name,* that is, the arbitrary application of names to things which are clearly designated by terms perfectly known; and it is of these alone that I speak.

Their utility and use is to elucidate and abbreviate discourse, in expressing by the single name that has been imposed what could otherwise be only expressed by several terms; so that nevertheless the name imposed remains divested of all other meaning, if it has any, having no longer any than that for which it is alone designed. Here is an example:

If we are under the necessity of discriminating numbers that are divisible equally by two from those which are not, in order to avoid the frequent repetition of this condition, a name is given to it in this manner: I call every number divisible equally by two, *an even number.*

This is a geometrical definition; because after having clearly designated a thing, namely, every member divisible equally by two, we give it a name divested of every other meaning, if it has any, in order to give it that of the thing designated.

Hence it appears that definitions are very arbitrary, and that they are never subject to contradiction; for nothing is more permissible than to give to a thing which has been clearly designated, whatever name we choose. It is only necessary to take care not to abuse the liberty that we possess of imposing names, by giving the same to two different things.

Not that this may not be permissible, provided we do not confound the consequences, and do not extend them from the one to the other.

But if we fall into this error, we can oppose to it a sure and infallible remedy: that of mentally substituting the definition in the place of the thing defined, and of having the definition always so present, that every time we speak, for example, of an even number, we mean precisely that which is divisible into two equal parts, and that these two things should be in such a degree joined and inseparable in thought, that as soon as the discourse expresses the one, the mind attaches it immediately to the other. For geometricians, and all those who proceed methodically, only impose names on things to abbreviate discourse, and not to diminish or change the idea of the things of which they are discoursing. And they pretend that the mind always supplies the full definition to the concise terms, which they only employ to avoid the confusion occasioned by the multitude of words.

Nothing more promptly and more effectually removes the captious cavils of sophists than this method, which it is necessary to have always present, and which alone suffices to banish all kinds of difficulties and equivocations.

These things being well understood, I return to the explanation of the true order, which consists, as I have said, in defining every thing and in proving every thing.

This method would certainly be beautiful, but it is abso-lutely impossible; for it is evident that the first terms that we wished to define would imply precedents to serve for their explanation, and that in the same manner, the first propositions that we wished to prove would imply others which had preceded them; and thus it is clear that we should never reach the first.

Thus, in pushing our researches further and further, we arrive necessarily at primitive words which can no longer be defined, and at principles so clear that we can find no others that can serve as a proof of them.

Hence it appears that men are naturally and immutably impotent to treat of any science so that it may be in an absolutely complete order.

But it does not thence follow that we should abandon every kind of order.

For there is one, and it is that of geometry, which is in truth inferior in that it is less convincing, but not in that it less is certain. It does not define every thing and does not prove every thing, and it is in this that it is inferior; but it assumes nothing but things clear and constant by natural enlightenment, and this is why it is perfectly true, nature sustaining it in default of discourse.

This order, the most perfect of any among men, consists not at all in defining every thing or in demonstrating every thing, nor in defining nothing or in demonstrating nothing, but in adhering to this middle course of not defining things clear and understood by all mankind, and of defining the rest; of not proving all the things known to mankind, and of proving all the rest. Against this order those sin alike who undertake to define everything and to prove every thing, and who neglect to do it in those things which are not evident of themselves.

This is what is perfectly taught by geometry. She does not define any of these things, *space, time, motion, number, equality,* and similar things which exist in great number, because these terms so naturally designate the things that they mean, to those who understand the language, that their elucidation would afford more obscurity than instruction.

For there is nothing more feeble than the discourse of those who wish to define these primitive words. What neces-sity is there, for example, of explaining what is understood by the word *man?* Do we not know well enough what the thing is that we wish to designate by this term? And what advantage did Plato think to procure us in saying that he was a two-legged animal without feathers? As though the idea that I have of him naturally, and which I cannot express, were not clearer and surer than that which he gives me by his useless and even ridiculous explanation; since a man does not lose humanity by losing the two legs, nor does a capon acquire it by losing his feathers.

There are those who are absurd enough to explain a word by the word itself. I know some who have defined light in this wise: *Light is a luminary movement of luminous bodies,* as though we could understand the words *luminary* and *luminous* without the word light.[2]

We cannot undertake to define being without falling into the same absurdity: for we cannot define a word without beginning with the word *it is,* either expressed or understood. To define being therefore, it is necessary to say *it is,* and thus to employ the word defined in the definition.

We see clearly enough from this that there are some words incapable of being defined; and, if nature had not supplied this defect by a corresponding idea which she has given to all mankind, all our expressions would be confused; whilst we use them with the same assurance and the same certainty as though they were explained in a manner perfectly exempt from ambiguities: because nature herself has given us, without words, a clearer knowledge of them than art could acquire by our explanations.

It is not because all men have same the idea of the essence of the things that I say that it is impossible and useless to define.

For, for example, time is of this sort. Who can define it? And why undertake it, since all men conceive what is meant in speaking of a time, without any further definition? Nevertheless there are many different opinions touching the essence of time. Some say that it is the movement of created thing; others, the measure of the movement, etc. Thus it is not the nature of these things that I say is known to all; it is simply the relation between the name and the thing; so that at the expression *time,* all direct their thoughts towards the same object; which suffices to cause this term to have no need of being defined, though afterwards, in examining what time is, we come to differ in sentiment after having been led to think of it; for definitions are only made to designate the things that are named, and not to show the nature of them.

It is not because it is not permissible to call by the name of *time* the movement of a created thing; for, as I have just said, nothing is more arbitrary than definitions.

But after this definition there will be two things that will be called by the name of *time:* the one is what the whole world understands naturally by this word and what all those who speak our language call by this term; the other will be the movement of a created thing, for this will also called by his name, according to this new definition.

It is necessary therefore to shun ambiguities and not to confound consequences. For it will not follow from this that the thing that is naturally understood by the word time is in fact the movement of a created thing. It has been allowable to name these two things the same; but it will not be to make them agree in nature as well as in name.

Thus, if we advance this proposition — *time is the movement of a created thing,* it is necessary to ask what is meant by this word *time,* that is, whether the usual and generally received meaning is left to it, or whether it is divested of this meaning in order to give to it on this occasion that of the movement of a created thing. For if it be stripped of all other meaning, it cannot be contradicted, and it will become an arbitrary definition, in consequence of which, as I have said, there will be two things that will have the same name. But if its ordinary meaning be left to it, and it be pretended nevertheless that what is meant by this word is the movement of a created thing, it can be contradicted. It is no longer an arbitrary definition, but a proposition that must be proved, if it is not evident of itself; and this will then be a principle or an axiom, but never a definition, since in this enunciation it is not understood that the word *time* signifies the same thing as the *movement of a created thing,* but it is understood that what is conceived by the term *time* is this supposed movement.

If I did not know how necessary it is to understand this perfectly, and how continually occasions like this, of which I give the example, happen both in familiar and scientific discourses, I should not dwell upon it. But it seems to me, by the experience that I have had from the confusion of controversies, that we cannot too fully enter into this spirit of precision, for the sake of which I write this treatise rather than the subject of which I treat in it.

For how many persons are there who fancy that they have defined time, when they have said that it is the measure of movement, leaving it, however, its ordinary meaning! And nevertheless they have made a proposition and not a definition. How many are there, in the like manner, who fancy that they have defined movement, when they have said: *Motus nec simpliciter motus, non mera potentia est, sed actus entis in potentia!* And nevertheless, if they leave to the word *movement* its ordinary meaning as they do, it is not a definition but a proposition; and confounding thus the definitions which they call *definitions of name,* which are the true arbitrary definitions permissible and geometrical, with those which they call *definitions of thing,* which, properly speaking, are not at all arbitrary definitions but are subject to contradiction, they hold themselves at liberty to make these as well as others; and each defining the same things in his own way, by a liberty which is as unjustifiable in this kind of definitions as it is permissible in the former, they perplex every thing, and losing all order and all light, become lost themselves and wander into inextricable embarrassments.

We shall never fall into such in following the order of geometry. This judicious science is far from defining such primitive words as *space, time motion, equality, majority, diminution, whole,* and others which every one understands. But apart from these, the rest of the terms that this science employs are to such a degree elucidated and defined that we have no need of a dictionary to understand any of them; so that in a word all these terms are perfectly intelligible, either by natural enlightenment or by the definitions that it gives of them.

This is the manner in which it avoids all the errors that may be encountered upon the first point, which consists in defining only the things that have need of it. It makes use of it in the same manner in respect to the other point, which consists in proving the propositions that are not evident.

For, when it has arrived at the first known truths, it pauses there and asks whether they are admitted, having nothing clearer whereby to prove them; so that all that is proposed by geometry is perfectly demonstrated, either by natural enlightenment or by proofs.

Hence it comes that if this science does not define and demonstrate every thing, it is for the simple reason that this is impossible.[3]

It will perhaps be found strange that geometry does not define any of the things that it has for its principal objects; for it can neither define motion, numbers, nor space; and nevertheless these three things are those of which it treats in particular, and according to the investigation of which it takes the three different names of *mechanics, arithmetic,* and *geometry,* this last name belonging to the genus and species.

But this will not surprise us if we remark that, this admirable science only attaching itself to the simplest things, this same quality which renders them worthy of being its objects renders them incapable of being defined; so that the lack of definition is a perfection rather than a defect, since it does not come from their obscurity, but on the contrary from their extreme obviousness, which is such that though it may not have the conviction of demonstrations, it has all their certainty. It supposes therefore that we know what is the thing that is understood by the words *motion, number, space;* and without stopping to define them to no purpose, it penetrates their nature and discovers their marvellous properties.

These three things which comprehend the whole universe, according to the words: *Deus fecit omnia in pondere, in numero, et mensura,* [4] have a reciprocal and necessary connection. For we cannot imagine motion without something that moves; and this thing being one, this unity is the origin of all numbers; and lastly, motion not being able to exist without space, we see these three things included within the first.

Time even is also comprehended in it; for motion and time are relative to each other; speed and slowness, which are the differences of motion, having a necessary relation to time.

Thus there are properties common to all these things, the knowledge of which opens the mind to the greatest marvels of nature.

The chief of these comprehends the two infinitudes which are combined in every thing: the one of greatness, the other of littleness.

For however quick a movement may be, we can conceive of one still more so; and so on *ad infinitum,* without ever reaching one that would be swift to such a degree that nothing more could be added to it. And, on the contrary, however slow a movement may be, it can be retarded still more; and thus *ad infinitum,* without ever reaching such a degree of slowness that we could not thence descend into an infinite number of others, without falling into rest.

In the same manner, however great a number may be, we can conceive of a greater; and thus *ad infinitum,* without ever reaching one that can no longer be increased. And on the contrary, however small a number may be as, the hundredth or ten thousandth part, we can still conceive of a less; and so on *ad infinitum,* without ever arriving at zero or nothingness.

However a great space may be, we can conceive of a greater; and thus *ad infinitum,* without ever arriving at one which can no longer be increased. And, on the contrary, however, small a space may be, we can still imagine a smaller; and so on *ad infinitum,* without ever arriving at one indivisible, which has no longer any extent.

It is the same with time. We can always conceive of a greater without an ultimate, and of a less without arriving at a point and a pure nothingness of duration.

That is, in a word, whatever movement, whatever number, whatever space, whatever time there may be, there is always a greater and a less than these: so that they all stand betwixt nothingness and the infinite, being always infinitely distant from these extremes.

All these truths cannot be demonstrated; and yet they are the foundations and principles of geometry. But as the cause that renders them incapable of demonstration is not their obscurity, but on the contrary their extreme obviousness, this lack of proof is not a defect, but rather a perfection.

From which we see that geometry can neither define objects nor prove principles; but for this single and advantageous reason that both are in an extreme natural clearness, which convinces reason more powerfully than discourse.

For what is more evident than this truth, that a number, whatever it may be, can be increased — can be doubled? Again, may not the speed of a movement be doubled, and may not a space be doubled in the same manner?

And who too can doubt that a number, whatever it may be, may not be divided into a half, and its half again into another half? For would this half be a nothingness? And would these two halves, which would be two zeros, compose a number?

In the same manner, may not a movement, however slow it may be, be reduced in speed by a half, so that it will pass over the same space in double the time, and this last movement again? For would this be a perfect rest? And would these two halves of velocity, which would be two rests, compose again the first velocity?

Lastly, may not a space, however small it may be, be divided into two, and these halves again? And how could these two halves become indivisible without extent, which joined together made the former extent?

There is no natural knowledge in mankind that precedes this, and surpasses it in clearness. Nevertheless, in order that there may be examples for every thing, we find minds excellent in all things else, that are shocked by these infinities and can in no wise assent to them.

I have never known any person who thought that a space could not be increased. But I have seen some, very capable in other respects, who affirmed that a space could be divided into two indivisible parts, however absurd the idea may seem.

I have applied myself to investigating what could be the cause of this obscurity, and have found that it chiefly consisted in this, that they could not conceive of a continuity divisible *ad infinitum,* whence they concluded that it was not divisible.

It is an infirmity natural to man to believe that he possesses truth directly; and thence it comes that he is always disposed to deny every thing that is incomprehensible to him; whilst in fact he knows naturally nothing but falsehood, and whilst he ought to receive as true only those things the contrary of which appear to him as false.

And hence, whenever a proposition is inconceivable, it is necessary to suspend the judgment on it and not to deny it from this indication, but to examine its opposite; and if this is found to be manifestly false, we can boldly affirm the former, however incomprehensible it may be. Let us apply this rule to our subject.

There is no geometrician that does not believe space divisible *ad infinitum*. He can no more be such without this principle than man can exist without a soul. And nevertheless there is none who comprehends an infinite division; and he only assures himself of this truth by this one, but certainly sufficient reason, that he perfectly comprehends that it is false that by dividing a space we can reach an indivisible part, that, is, one that has no extent.

For what is there more absurd than to pretend that by continually dividing a space, we shall finally arrive at such a division that on dividing it into two, each of the halves shall remain indivisible and without any extent, and that thus these two negations of extensions will together compose an extent? For I would ask those who hold this idea, whether they conceive clearly two indivisibles being brought into contact; if this is throughout, they are only the same thing, and consequently the two together are indivisible; and if it is not throughout, it is then but in a part; then they have parts, therefore they are not indivisible.

If they confess, as in fact they admit when pressed, that their proposition is as inconceivable as the other, they acknowledge that it is not by our capacity for conceiving these things that we should judge of their truth, since these two contraries being both inconceivable, it is nevertheless necessarily certain that one of the two is true.

But as to these chimerical difficulties, which have relation only to our weakness, they oppose this natural clearness and these solid truths: if it were true that space was composed of a certain finite number of indivisibles, it would follow that two spaces, each of which should be square, that is, equal and similar on every side, being the one the double of the other, the one would contain a number of these indivisibles double the number of the indivisibles of the other. Let them bear this consequence well in mind, and let them then apply themselves to ranging points in squares until they shall have formed two, the one of which shall have double the points of the other; and then I will make every geometrician in the world yield to them. But if the thing is naturally impossible, that is, if it is an insuperable impossibility to range squares of points, the one of which shall have double the number of the other, as I would demonstrate on the spot did the thing merit that we should dwell on it, let them draw therefrom the consequence.

And to console them for the trouble they would have in certain junctures, as in conceiving that a space may have an infinity of divisibles, seeing that these are run over in so little time during which this infinity of divisibles would be run over, we must admonish them that they should not compare things so disproportionate as is the infinity of divisibles with the little time in which they are run over: but let them compare the entire space with the entire time, and the infinite divisibles of the space with the infinite moments of the time; and thus they will find that we pass over an infinity of divisibles in an infinity of moments, and a little space in a little time; in which there is no longer the disproportion that astonished them.

Lastly, if they find it surprising that a small space has as many parts as a great one, let them understand also that they are smaller in measure, and let them look at the firmament through a diminishing glass, to familiarize themselves with this knowledge, by seeing every part of the sky in every part of the glass.

But if they cannot comprehend that parts so small that to us they are imperceptible, can be divided as often as the firmament, there is no better remedy than to make them look through glasses that magnify this delicate point to a prodigious mass; whence they will easily conceive that by the aid of another glass still more artistically cut, they could be magnified so as to equal that firmament the extent of which they admire. And thus these objects appearing to them now easily divisible, let them remember that nature can do infinitely more than art.

For, in fine, who has assured them that these glasses change the natural magnitude of these objects, instead of re-establishing, on the contrary, the true magnitude which the shape of our eye may change and contract like glasses that diminish?

It is annoying to dwell upon such trifles; but there are times for trifling.

It suffices to say to minds clear on this matter that two negations of extension cannot make an extension. But as there are some who pretend to elude this light by this marvellous answer, that two negations of extension can as well make an extension as two units, neither of which it is a number, can make a number by their combination; it is necessary to reply to them that they might in the same manner deny that twenty thousand men make an army, although no single one of them is an army; that a thousand houses make a town, although no single one is a town; or that the parts make the whole, although no single one is the whole; or, to remain in the comparison of numbers, that two binaries make a quaternary, and ten tens a hundred, although no single one is such.

But it is not to have an accurate mind to confound by such unequal comparisons the immutable nature of things with their arbitrary and voluntary names, names dependent upon the caprice of the men who invented them. For it is clear that to facilitate discourse the name of *army* has been given to twenty thousand men, that of *town* to several houses, that of *ten* to ten units; and that from this liberty spring the names of *unity, binary, quaternary, ten, hundred,* different through our caprices, although these things may be in fact of the same kind by their unchangeable nature, and are all proportionate to each other and differ only in being greater or less, and although, as a result of these names, binary may not be a quaternary, nor the house a town, any more than the town is a house. But again, although a house is not a town, it is not however a negation of a town; there is a great difference between not being a thing, and being a negation of it.

For, in order to understand the thing to the bottom, it is necessary to know that the only reason why unity is not in the ranks of numbers, is that Euclid and the earliest authors who treated of arithmetic, having several properties to give that were applicable to all the numbers except unity, in order to avoid often repeating that *in all numbers except unity this condition is found,* have excluded unity from the signification of the word *number,* by the liberty which we have already said can be taken at will with definitions. Thus, if they had wished, they could in the same manner have excluded the binary and ternary, and all else that it pleased them; for we are master of these terms, provided we give notice of it; as on the contrary we may place unity when we like in the rank of numbers, and fractions in the same manner. And, in fact, we are obliged to do it in general propositions, to avoid saying constantly, that *in all numbers, as well as in unity and in fractions, such a property is found;* and it is in this indefinite sense that I have taken it in all that I have written on it.

But the same Euclid who has taken away from unity the name of number, which it was permissible for him to do, in order to make it understood nevertheless that it is not a negation, but is on the contrary of the same species, thus defines homogeneous magnitudes: *Magnitudes are said to be of the same kind, when one being multiplied several times may exceed the other;* and consequently, since unity can, be-ing multiplied several times, exceed any number whatsoever, it is precisely of the same kind with numbers through its essence and its immutable nature, in the meaning of the same Euclid who would not have it called a number.

It is not the same thing with an indivisible in respect to an extension. For it not only differs in name, which is voluntary, but it differs in kind, by the same definition; since an indivisible, multiplied as many times as we like, is so far from being able to exceed an extension, that it can never form any thing else than a single and exclusive indivisible; which is natural and necessary, as has been already shown. And as this last proof is founded upon the definition of these two things, indivisible and extension, we will proceed to finish and perfect the demonstration.

An indivisible is that which has no part, and extension is that which has divers separate parts.

According to these definitions, I affirm that two indivisibles united do not make an extension.

For when they are united, they touch each other in some part; and thus the parts whereby they come in contact are not separate, since otherwise they would not touch each other. Now, by their definition, they have no other parts; therefore they have no separate parts; therefore they are not an extension by the definition of extension which involves the separation of parts.

The same thing will be shown of all the other indivisibles that may be brought into junction, for the same reason. And consequently an indivisible, multiplied as many times as we like, will not make an extension. Therefore it is not of the same kind as extension, by the definition of things of the same kind.

It is in this manner that we demonstrate that indivisibles are not of the same species as numbers. Hence it arises that two units may indeed make a number, because they are of the same kind; and that two indivisibles do not make an extension, because they are not of the same kind.

Hence we see how little reason there is in comparing the relation that exists between unity and numbers with that which exists between indivisibles and extension.

But if we wish to take in numbers a comparison that rep-resents with accuracy what we are considering in extension, this must be the relation of zero to numbers; for zero is not of the same kind as numbers, since, being multiplied, it cannot exceed them: so that it is the true indivisibility of number, as indivisibility is the true zero of extension. And a like one will be found between rest and motion, and between an instant and time; for all these things are heterogeneous in their magnitudes, since being infinitely multiplied, they can never make any thing else than indivisibles, any more than the indivisibles of extension, and for the same reason. And then we shall find a perfect correspondence between these things; for all these magnitudes are divisible ad infinitum, without ever falling into their indivisibles, so that they all hold a middle place between infinity and nothingness.

Such is the admirable relation that nature has established between these things, and the two marvellous infinities which she has proposed to mankind, not to comprehend, but to admire; and to finish the consideration of this by a last remark, I will add that these two infinites, although infinitely different, are notwithstanding relative to each other, in such a manner that the knowledge of the one leads necessarily to the knowledge of the other.

For in numbers, inasmuch as they can be continually augmented, it absolutely follows that they can be continually diminished, and this clearly; for if a number can be multiplied to 100,000, for example, 100,000th part can also be taken from it, by dividing it by the same number by which it is multiplied; and thus every term of augmentation will become a term of division, by changing the whole into a fraction. So that infinite augmentation also includes necessarily infinite division.

And in space the same relation is seen between these two contrary infinites; that is, that inasmuch as a space can be infinitely prolonged, it follows that it may be infinitely diminished, as appears in this example: If we look through a glass at a vessel that recedes continually in a straight line, it is evident that any point of the vessel observed will continually advance by a perpetual flow in proportion as the ship recedes. Therefore if the course of the vessel is extended *ad infinitum,* this point will continually recede; and yet it will never reach that point in which the horizontal ray carried from the eye to the glass shall fall, so that it will constantly approach it without ever reaching it, unceasingly dividing the space which will remain under this horizontal point without ever arriving at it. From which is seen the necessary conclusion that is drawn from the infinity of the extension of the course of the vessel to the infinite and infinitely minute division of this little space remaining beneath this horizontal point.

Those who will not be satisfied with these reasons, and will persist in the belief that space is not divisible *ad infinitum,* can make no pretensions to geometrical demonstrations, and although they may be enlightened in other things, they will be very little in this; for one can easily be a very capable man and a bad geometrician.

But those who clearly perceive these truths will be able to admire the grandeur and power of nature in this double infinity that surrounds us on all sides, and to learn by this marvellous consideration to know themselves, in regarding themselves thus placed between infinitude and a negation of extension, between an infinitude and a negation of number, between an infinitude and a negation of movement, between an infinitude and a negation of time. From which we may learn to estimate ourselves at our true value, and to form reflections which will be worth more than all the rest of geometry itself.

I have thought myself obliged to enter into this long discussion for the benefit of those who, not comprehending at first this double infinity, are capable of being persuaded of it. And although there may be many who have sufficient enlightenment to dispense with it, it may nevertheless happen that this discourse which will be necessary to the one will not be entirely useless to the other.

---

1. After this paragraph occur in the MS. the following lines, written in a finer hand, and inclosed in parenthesis:

> ". . . is much more to succeed in the one than the other, and I have chosen this science to attain it only because it alone knows the true rules of reasoning, and, without stopping at the rules of syllogisms which are so natural that we cannot be ignorant of them, stops and establishes itself upon the true method of conducting reasoning in all things, which almost every one is ignorant of, and which it is so advantageous to know, that we

see by experience that among equal minds and like circumstances, he who possesses geometry bears it away, and acquires a new vigor.

"I wish, therefore, to explain what demonstrations are by the example of those of geometry, which is almost the only one of the human sciences that produces infallible ones, because she alone observes the true method, whilst all the others are, through a natural necessity, in a sort of confusion, which the geometricians alone know exceedingly well how to comprehend."

On the margin of this fragment is in the MS. the following note: "That which is in small characters was hidden under a paper, the edges of which were glued, and upon which was written the article beginning: I cannot better explain, etc."— Faugère.

2. Pascal alludes here to Father Noël, a Jesuit, with whom he had had a warm discussion on the subject of his *Expériences touchant le vide.* In a letter that he wrote to Father Noël in 1647, he said: "The sentence which precedes your closing compliments defines light in these terms: *Light* is a *luminous motion of rays composed of lucid, that is, luminous bodies;* upon which, I have to tell you that it seems to me that you ought first to have defined what *luminous* is, and what a *lucid* or *luminous body* is, for till then, I cannot understand what light is. And as we never make use in definitions of the term of the *thing defined,* I should have difficulty in conforming to yours which says: Light is a luminary motion of a luminous body."— Faugère.

3. Here the Ms. adds in parenthesis: "(But as nature punishes all that science does not bestow, its order in truth does not give a superhuman perfection, but it has all that man can attain. It has seemed to me proper to give from the beginning of this discourse this, etc.)."— Faugère.

4. "God has made all things in weight, number and proportion."